BAREME-SOLUTII-PROBLEME-OLIMPIADA DE MATEMATICA-FAZA LOCALA-14 FEBRUARIE 2014

1.a)Se observa ca $26 = 3 \cdot 9 - 1 ; 23 = 3 \cdot 8 - 1 ; \ 20 = 3 \cdot 7 - 1$ si asa mai departe avem ca $2 = 3 \cdot 1 - 1$, deci primii doi termeni ai sirului sunt 2 si 5(=3· $2 - 1$)

2 puncte .

b) termenul 20 este 3· $20 - 1 = 59$

S=2+5+8+…+59=(2+59)X20:2=610

1 puncte.

c) orice termen al sirului este de forma 3n-1 . Daca ar fi patrat perfect atunci $3n-1=p^2$, deci $3n=p^2+1$. Daca p=3k atunci $3n=(3k)^2+1$,deci $M_3=M_3+1$ fals , daca p=3k+1 atunci $3n=(3k+1)^2+1$,deci $M_3=M_3+2$ fals ,daca p=3k+2 atunci $3n=(3k+2)^2+1$,deci $M_3=M_3+5$ fals ,in concluzie nici un termen al sirului nu poate fi patrat perfect .

4 puncte.

2.a) (2,1,1) solutie daca x=2,y=1 si z=1 care verifica ecuatia . 1 punct.

$(2^8,2^4,2^2)$ solutie daca $x=2^8$,$y=2^4$ si $z=2^2$ verifica ecuatia ,

deci $2^8 \cdot 2^8+2 \cdot 2^8 \cdot 2^6=6 \cdot 2^8 \cdot 2^6 / : 2^{15}$, avem atunci 2+1=3 (A) 2 puncte.

b)Notam $x=2^a$, $y=2^b$, $z=2^c$, avem $2^{a+2b} +2^{1+a+3c} =3 \cdot 2^{1+2b+3c}$,luam relatiile : 1+a+3c=a+2b+1 si a+2b=1+2b+3c , deducem a=1+3c si 3c=2b , b=3c/2. Luam c=2n, a=6n+1 si b=3n . Obtinem ca $x=2^{6n+1}$, $y=2^{3n}$ si $z=2^{2n}$ care verifica ecuatia data pentru orice n numar natural , deci avem o infinitate de solutii .

4 puncte.

3.a)$\frac{m(\measuredangle COD)}{4} = \frac{m(\measuredangle BOC)}{3} = k \ \rightarrow m(\measuredangle BOC) = 3k \ si \ m(\measuredangle COD) = 4k \rightarrow m(\measuredangle AOB) = 3k + 10^0. Din \ 3 \cdot m(\measuredangle AOE) = 5 \cdot m(\measuredangle BOC) \ avem \ ca \ m(\measuredangle AOE) = 5k \ si \ m(\measuredangle DOE) = \frac{5k}{2} , apoi \ din \ suma \ lor \ avem \ ca$ k=20 atunci $m(\measuredangle AOB) = 70^0, m(\measuredangle BOC) = 60^0, m(\measuredangle COD) = 80^0, m(\measuredangle DOE) = 50^0, \ m(\measuredangle AOE = 100^0.$

5 puncte.

b)[OF bis. Atunci $m(\measuredangle AOF) = 50^0, m(\measuredangle AOC) = 70^0 + 60^0 = 130^0$ si $m(\measuredangle FOC) = 180^0$. Atunci F,O,C coliniare .

2 puncte.

4.Notam AB=x , AM=BM=x/2 , $MM_1=M_1B=x/4$, $AM_1=x/2+x/4=3x/4$, $AM_2=M_2M_1=3x/8$, $AM_3=3x/8+x/8+3x/16=11x/16$, $AM_4=11x/32=M_4M_3$, $M_2M=3x/8-x/4=x/8$, $M_2B=x/8+x/2=5x/8$, $M_2M_3=M_3B=5x/16$, $M_3M_1=5x/16-x/4=x/16$. (5x/16>x/8 , 5x/16<3x/8) , $MM_3=x/4-x/16=3x/16$.

3 puncte.

a)Avem atunci x/16=x/32+2 , deducem x=64 , AB=64 cm.

1 punct

b)$M_2M_a=M_aM_4=x/64$, $M_1M_b=M_bM_3=x/32$. $AM_a=11x/32+x/64=23x/64$ si $M_aM_b=x/64+x/8+3x/16+x/32=(x+8x+12x+2x)/64=23x/64$, deci M_a este mijlocul lui $[AM_b]$

3 puncte.